A new generation of design equations for reinforced elastomeric bearings

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ABSTRACT

Provisions for seismic isolation have recently been incorporated into the National Building Code of Canada and Canadian Highway Bridge Design Code. In addition to the global response of the structure, the design of the isolation devices is an important consideration for codes and standards. Reinforced elastomeric bearings are commonly used in seismic isolation applications and as bridge bearings. Relevant codes and standards have been developed to guide designers on several critical design properties, such as the compression and bending modulus and the maximum shear strain due to compression and rotation. These codes and standards commonly assume that the elastomer is incompressible and that the reinforcement is rigid and inextensible. These assumptions are most appropriate for reinforced elastomeric bearings with a low shape factor, defined as the ratio of the loaded area to unloaded area of a single layer of elastomer. Application for seismic isolation purposes generally require higher shape factors and retention of these assumptions can rapidly introduce significant error as the shape factor increases. In this paper, a new generation of design equations are proposed and discussed for the critical design properties of reinforced elastomeric bearings. The proposed design equations include the compressibility of the elastomer and the extensibility of the reinforcement. The derivation of the design equations is presented. The proposed design equations are compared against available analytical solutions and current design provisions. It is shown that the proposed design equations are an accurate representation of the analytical solution and significantly reduce the potential error in critical design properties.

Keywords: elastomeric bearing, base isolation, compressibility, extensibility, design equations

INTRODUCTION

Provisions for seismic base isolation have recently been included in the National Building Code of Canada (NBCC) [1] and the Canadian Highway Bridge Design Code (CHBDC) (CSA S6-14 [2]). It is anticipated that the inclusion of these provisions will increase the application of base isolation technology within Canada. As base isolation technology becomes more commonplace, it is necessary to develop increasingly comprehensive codes and standards to guide designers. It must be ensured that the device is properly designed and installed to achieve high performance in a seismic event. The NBCC does not specifically cover requirements for isolation device design, but rather focuses on global response and loading aspects (i.e. displacement demand, restoring force, base shear etc.). Although the CHBDC is not intended to apply for buildings, the commentary of the NBCC [3] recommends reviewing it for more information on seismic isolation. Unlike the NBCC, the CHBDC does cover specific requirements for elastomeric and sliding systems used for seismic isolation and in conventional bridge bearing applications.

Elastomeric isolators, similar to conventional bridge bearings, are composed of alternating horizontal layers of reinforcement and elastomer, as shown in Figure 1. The reinforcement is necessary to enhance the vertical properties of the isolator but otherwise has little impact on the horizontal properties. The result is a device that is stiff in the vertical direction but flexible in the lateral direction. The requirements for elastomeric devices in the CHBDC assume that the elastomer is incompressible and that the reinforcement is inextensible. These assumptions are most appropriate for isolators with a low shape factor, defined as the ratio of the loaded area to unloaded area of a single layer of elastomer. Applying these assumptions for larger shape factors may quickly lead to significant error as the shape factor increases [4–6]. Use of non-rigid (e.g. fiber) reinforcement has also recently been permitted by the CHBDC and should be considered as a design parameter.

In this paper, a new generation of design equations are proposed for elastomeric bearings that include the effects of the compressibility of the elastomer and the extensibility of the reinforcement. The proposed equations are mathematically derived from existing closed-form analytical solutions for critical design properties, such as the compression modulus, bending modulus, maximum shear strain due to compression and the maximum shear strain due to rotation. The derivation procedure is given and discussed. An optimization procedure is used to minimize the error compared to the analytical solution and the proposed equations are compared against the analytical solutions and current standards. Note that the terms isolator and bearing are used interchangeably.

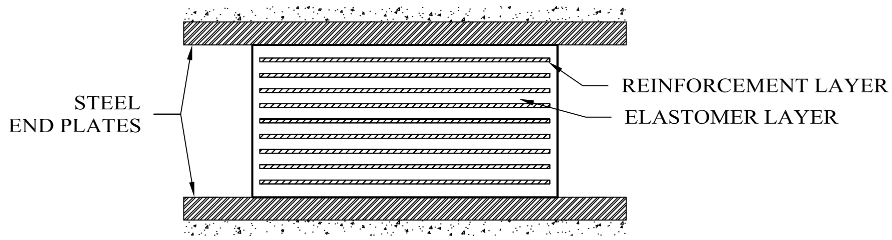


Figure 1: Typical Elastomeric Bearing (note: the steel end plates are not necessarily required)

CURRENT GUIDELINES AND STANDARDS

Canadian Highway Bridge Design Code

The CHBDC limits the total shear strain in elastomeric bearings due to compression, horizontal displacement and rotation under dead load, seismic load and thermal, creep, and shrinkage effects to:

$$\gamma_c + \gamma_r + \gamma_d \leq 5.5 \tag{1}$$

where γ_c is the maximum shear strain due to compression, γ_r is the maximum shear strain due to rotation, and γ_d is the maximum shear strain due to total design displacements. As shown in Figure 2, the maximum shear strain due to compression and rotation develops at the extreme corners of each layer of elastomer. The reinforcement restrains the elastomeric layers which, due to the near incompressibility of the elastomer and Poisson's effect, bulge laterally as a compressive force or rotation is applied. This composite action results in significantly stiffer compression and bending properties.

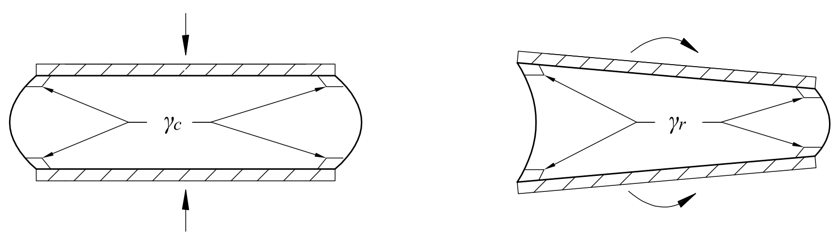


Figure 2: Location of the maximum shear stress due to compression and rotation for a single layer of elastomer between reinforcement layers

According to the CHBDC, γ_r is defined as:

$$\gamma_r = \frac{D_r B^2}{t_i T}\theta_t \tag{2}$$

where D_r is the *shape factor* for rotation (0.375 for a circular bearing and 0.55 for rectangular bearings), B is the plan dimension in the direction of loading (diameter for a circular bearing), θ_t is the total rotation of the isolator under dead loads and construction loads, t_i is the thickness of the ith layer of elastomer, and T is the total thickness of the elastomer.

Similarly, γ_c, is defined as:

$$\gamma_c = \frac{D_c \sigma_c}{GS} \tag{3}$$

where σ_c is the average vertical stress due to compression, G is the shear modulus of the elastomer, S is the shape factor of the elastomeric layers, and D_c is the *shape factor* for compression (1.0 for both circular and rectangular bearings). Note that the unfortunate terminology of D_r and D_c as '*shape factor'* used in the CHBDC should not be confused with S.

If incompressibility and inextensibility are assumed, it can be shown that for all basic pad geometries $E_c \propto GS^2$, $\gamma_c \propto S\varepsilon_c$ and $\gamma_r \propto S^2\theta$, where ε_c is the compressive strain and θ is the rotation of the elastomeric layer [7]. Thus, with $\varepsilon_c = \sigma_c/E_c$, and assuming all layers have equal thickness, the derivation of Eq. (2) and Eq. (3) is relatively simple and from Eq. (2) and Eq. (3) it is implicit in the CHBDC that the elastomer is incompressible and the reinforcement is inextensible.

The properties of a rectangular pad are dependent on the length-to-width aspect ratio, which is not accounted for in Eq. (2) or Eq. (3). For example, a square pad with an incompressible elastomer and inextensible reinforcement has a value of D_r = 0.47

and D_c = 1.2 derived from the analytical solutions. Therefore, for a square pad the CHBDC provisions (i.e. D_r = 0.55 and D_r = 1.0) are 17% conservative for rotation, and 17 % unconservative for compression. Figure 3 shows the ratio of the shear strain determined from the CHBDC to the analytical solution assuming an incompressible elastomer and inextensible reinforcement as a function of the aspect ratio, ρ. A value of $\rho > 1$ is representative of bending about the strong axis of the rectangular pad. The magnitude of the error ranges between 11 % and 49%, and 8 % and 27 %, for rotation and compression, respectively. Note that this significant error is present before the effects of compressibility or extensibility is included.

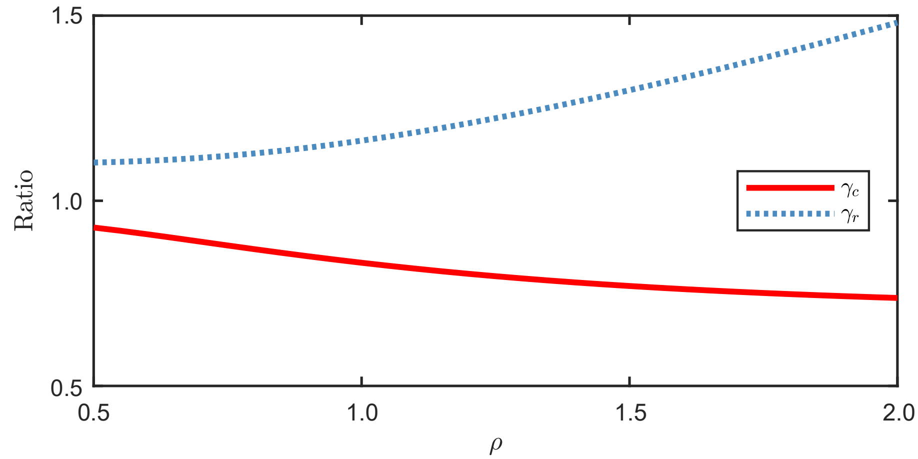


Figure 3: Ratio of the shear strain determined from the CHBDC to the analytical solution assuming an incompressible elastomer and inextensible reinforcement.

International Organization for Standardization

Although not explicitly included in the CHBDC, the compression modulus, E_c, and bending modulus, E_b, of the isolator may also be of interest to designers. Gent and Lindley [8] proposed that the compression and bending modulus including compressibility could be obtained by correcting the solution assuming incompressibility, E_c^∞ or E_b^∞, by the bulk modulus, K. This ad hoc approximation, which is currently used in ISO 22762 [9], is:

$$\frac{1}{E_c} = \frac{1}{E_c^\infty} + \frac{1}{K} \tag{4}$$

$$\frac{1}{E_b} = \frac{1}{E_b^\infty} + \frac{1}{K} \tag{5}$$

These approximations are intuitively based. They capture the correct limits of the compression and bending modulus, (e.g. for a small S, $E_c \to E_c^\infty$ and as $S \to \infty$, $E_c \to K$). The latter limit represents the convergence of the properties to the bulk modulus as the restraint on the lateral bulging of the elastomer approaches full confinement and the deformation becomes entirely dependent on volumetric strain.

PROPOSED DESIGN EQUATIONS

Derivation Example

Analytical solutions for the aforementioned critical design properties are available for most common pad geometries (e.g. [7,10–16]). These analytical solutions have been derived based on the assumptions of the pressure solution, which notably assumes that the lateral bulging of the elastomer follows a parabolic curve over the height of the elastomeric layer, horizontal planes remain plane and the elastomer is linear elastic. The procedure used to derive the proposed design equations was originally applied by Chalhoub and Kelly [17]. It has since been refined and applied to a broader range of pad geometries and critical design properties by Van Engelen and Kelly [6] and Van Engelen et al. [4,5]. The derivation procedure is demonstrated through an example for the compression modulus of an infinite strip pad.

The analytical solution for the compression modulus of an infinite strip pad, derived based on the assumptions of the pressure solution is [10]:

$$E_c = \frac{12GS^2}{\lambda^2}\left(1 - \frac{1}{\lambda}\tanh(\lambda)\right) \tag{6}$$

where

$$\lambda^2 = \frac{12GS^2}{K_e(1)} \tag{7}$$

and

$$\frac{1}{K_e(e)} = \frac{1}{K} + e\frac{t}{E_f t_f} \tag{9}$$

The dimensionless parameter λ appears in all analytical solutions derived based on the assumptions of pressure solution that include the extensibility of the reinforcement and/or compressibility of the elastomer. The coefficient of the right-hand-side of Eq. (7) (i.e. 12 for the compression modulus of an infinite strip pad) is dependent on the pad geometry and critical design parameter considered. The variable K_e is an equivalent parameter that accounts for the compressibility of the elastomer and the extensibility of the reinforcement. In Eq. (9) $e = 1$ for an infinite strip pad, this is likewise dependent on the pad geometry and critical design parameter considered.

The Taylor series expansion of Eq. (6) yields:

$$E_c = K_e(1)\left(\frac{1}{3}\lambda^2 - \frac{2}{15}\lambda^4 + \frac{17}{315}\lambda^6 + O(\lambda^8)\right) \tag{10}$$

Substituting in the definition of λ from Eq. (10) and truncating the series yields:

$$E_c = 4GS^2\left(1 - \frac{24}{5}\frac{GS^2}{K_e(1)}\right) \tag{11}$$

Inverting gives:

$$\frac{1}{E_c} = \frac{1}{4GS^2}\left(1 - \frac{24}{5}\frac{GS^2}{K_e(1)}\right)^{-1} \tag{12}$$

Expanding with a binomial series:

$$\frac{1}{E_c} = \frac{1}{4GS^2} + \frac{6}{5}\frac{1}{K_e(1)} \tag{13}$$

Equation (13) is the approximation initially proposed by Van Engelen et al. [5]. The denominator of the first term on the right-hand-side is the analytical solution assuming an incompressible elastomer and inextensible reinforcement (i.e. E_c^{∞}). The second term, similar to the ad hoc approximation by Gent and Lindley [8], corrects for compressibility and extensibility.

Accuracy

Consider the Taylor series expansion and the inverted Taylor series expansion of Eq. (6):

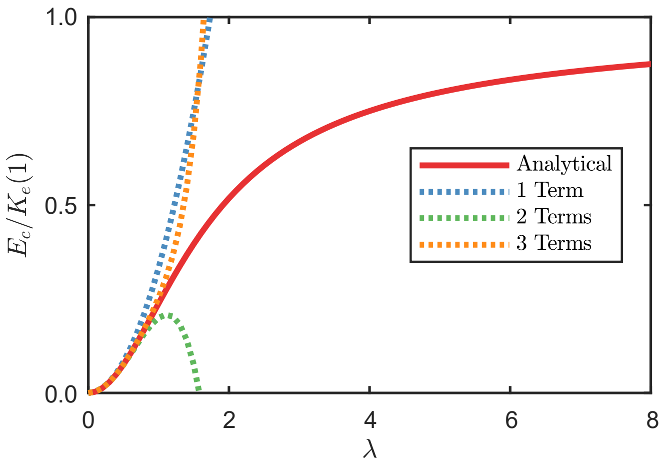
$$\frac{E_c}{K_e(1)} = \frac{1}{3}\lambda^2 - \frac{2}{15}\lambda^4 + \frac{17}{315}\lambda^6 + O(\lambda^8) \tag{14}$$

and

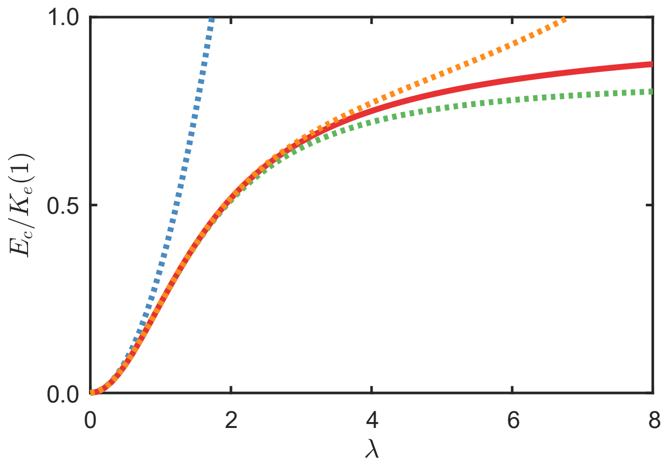
$$\frac{K_e(1)}{E_c} = 3\frac{1}{\lambda^2} + \frac{6}{5} - \frac{1}{175}\lambda^2 + O(\lambda^4) \tag{15}$$

A Taylor series expansion typically becomes divergent at some value of λ regardless of how many terms are taken. Figure 4 compares Eq. (14) and Eq. (15) assuming the series have been truncated to one, two or three-terms. The expansion of Eq. (14) quickly becomes divergent from the analytical solution in all cases considered. However, the two-term truncation of Eq. (15)

accurately follows the analytical solution over the entire range of λ, although the one and three-term truncations are divergent. Note that the form of the two-term truncation closely resembles the ad hoc approximation. The order of the error in the inverted truncated two-term expansion, 2, is less than the original expansion, 6. Therefore the error of the inverted term is of a lower order and expected to result in a better approximation. Furthermore, the coefficient of the third term is an order of magnitude lower in the inverted expansion, indicating that the error will be less in the inverted two-term truncation. The sign of the third term determines that the inverted expansion will always be conservatively lower than the analytical solution. The result is a maximum error of 10% over the range of λ considered.



(a)



(b)

Figure 4: Comparison of the analytical solution for E_c to the truncated Taylor series expansion of the (a) original expression and (b) inverted expression.

PROPOSED EQUATIONS FOR DESIGN

Generalized Equations

A similar procedure as described above can be applied for the different pad geometries and critical design properties with some variations. The result is simple design equations that can be corrected to any pad geometry by tabulated geometry specific correction factors. The general form of these approximations is:

$$\frac{1}{E} = \frac{1}{\Gamma_1 GS^2} + \frac{\Gamma_2}{K_e(e)} \tag{16}$$

$$\frac{\sigma_c}{\gamma_c GS} \vee \frac{\gamma_r}{\theta S^2} = \begin{cases} \Gamma_3 - \dfrac{\lambda^2}{\Gamma_4}, \text{ if } \lambda \le \lambda_t \\ \dfrac{\Gamma_5}{\lambda} - \dfrac{\Gamma_6}{\lambda^2}, \text{if } \lambda > \lambda_t \end{cases} \tag{17}$$

where

$$\lambda^2 = \frac{cGS^2}{K_e(e)} \tag{18}$$

E is the compression or bending modulus and Γ represents geometry and loading specific coefficients.

Note that the general format for the maximum shear strain due to compression and rotation are inverses of each other, as shown in Eq. (17). This is indicative of some of the challenges in elastomeric bearing design where an increase in S vertically stiffens the bearing, reducing γ_c. However, it also increases the rotational stiffness, resulting in an increased value of γ_r for the same level of rotation. Since the rotational stiffness of a bearing is expected to be significantly lower than the supports (e.g. a bridge girder), the increased rotational stiffness of the bearing does not significantly impact the applied rotation.

Optimization

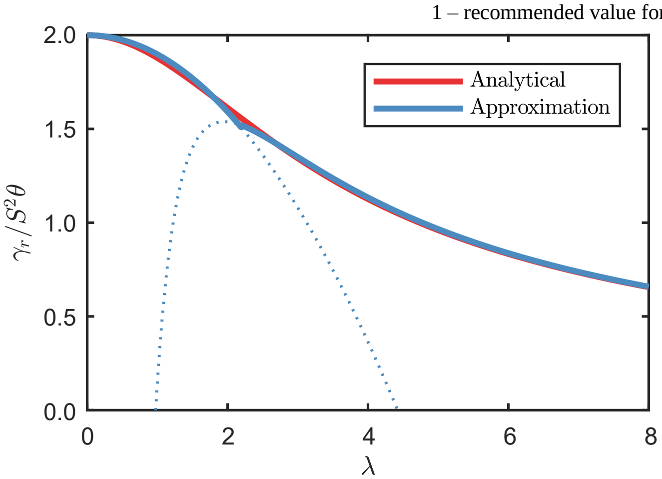
In general, the proposed approximations in Ref. [4–6] were derived directly from the analytical solution as shown above. While the mathematically derived expression is convenient, it is not necessarily the most accurate expression for a selected range of parameters. It is possible to optimize the proposed design equations by minimizing the squared residuals over a selected range of λ. For the compression and bending modulus Γ_2 was optimized. For the maximum shear strain due to compression and rotation Γ_4, Γ_6, and λ_t were simultaneously optimized. The selected optimized parameters were determined based on the shape of the generalized expressions. The maximum shear strain is fitted with two functions that transition at a λ_t.

The results of the optimization procedure are provided in Table 1. Figure 5 compares the maximum shear strain due to compression and rotation of the proposed design equations to the analytical solutions for an infinite strip pad. The figure shows the divergence of the two proposed approximations in the dotted line. The transition between the two expressions can be seen by an instantaneous change in curvature. The magnitude of the error of the optimized solutions for an infinite strip pad and circular isolator does not exceed 6.2% over the range of λ considered for any critical design parameter. Therefore, the proposed design equations are highly accurate when compared to the respective analytical solution.

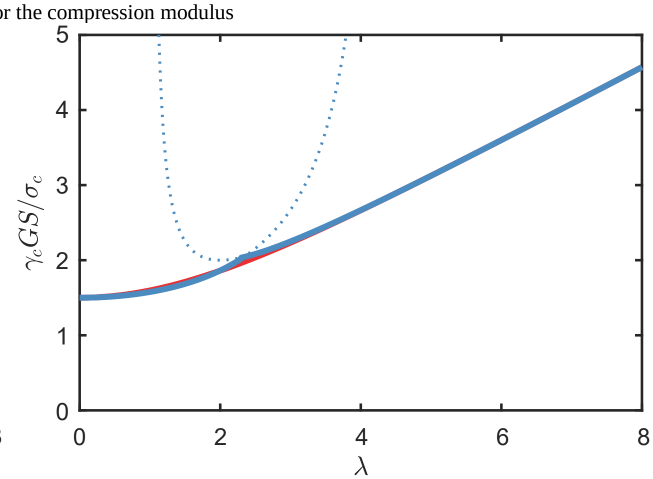
Table 1: Tabulated Coefficients for the Proposed Design Equations

Property/Geometry		Coefficient								
		c	Γ_1	Γ_2	Γ_3	Γ_4	Γ_5	Γ_6	e	λ_t
Compression	Infinite Strip	12	4	1.13	2/3	30.8	2	2	1	2.3
	Circular Pad	48	6	1.21	1	37.0	4	5.71	1 (7/4)[1]	3.4
Rotation	Infinite Strip	12	4/5	1.30	2	9.78	6	5.85	1	2.2
	Circular Pad	48	2	1.26	6	5.54	24	32.7	-	3.0

1 – recommended value for the compression modulus



(a)



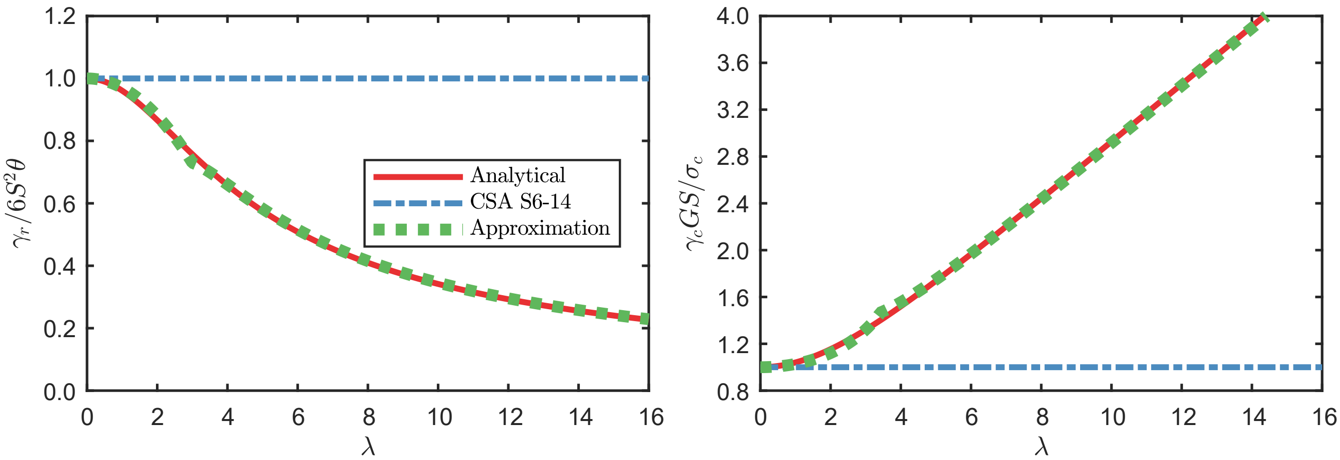
(b)

Figure 5: Comparison of the analytical solution to the proposed approximation for (a) γ_r and (b) γ_c of an infinite strip pad

Example: Circular Pad

Figure 6 compares the analytical solutions to the proposed design equations and the CHBDC equations as a function of λ. Note the CHBDC equations do not account for compressibility or extensibility and are consequently independent of λ. It is evident that significant error may be introduced by using the CHBDC expressions if the effects of compressibility and extensibility are ignored and λ is large (i.e. approximately greater than 1.0).

The bulk modulus of elastomers is a difficult material property to measure. In absence of better information, AASHTO LRFD Bridge Design Specification [18] recommends the assumption that K = 3100 MPa. Assuming a typical value of G = 0.5 MPa, inextensible reinforcement and a low shape factor of S = 5, λ = 0.44 for a circular pad. Under these assumptions, the magnitude of the error is negligible (approximately 1% in both cases) and the use of the CHBDC equations is appropriate. Relaxing the assumption of inextensible reinforcement and selecting a value of E_f = 20,000 MPa and t_f/t = 0.1 (representative of fiber reinforcement) increases the error to approximately 2% and the CHBDC expressions remain appropriate. However, increasing S from 5 to 10 increases the magnitude of the error to approximately 8%. An S of 20 would have an error of 28 % and 22 %, for γ_r and γ_c, respectively (λ = 2.8).



(a) (b)

Figure 6: Comparison of the analytical solution to the proposed approximation and the CHBDC equation for (a) γ_r and (b) γ_c of a circular pad

DISCUSSION

The range of λ considered in the figures is based on: $5 \leq S \leq 50$; $1000 \leq K/G \leq 10{,}000$; and $3000 \leq E_f t_f/tG \leq 60{,}000$ (or approximately $500 \leq K_e/G \leq 10{,}000$). The case of $\lambda \rightarrow 0$ is representative of an incompressible elastomer and inextensible reinforcement. Early development of the non-optimized proposed approximation [4–6] often found the magnitude of the error increases with increasing λ. The approach presented herein, with the use of optimization and revision of the shear equation from what was originally presented in Van Engelen et al. [5], has determined that the proposed design equations can be accurate over a broad range of λ. However, it is expected that most practical cases will fall within the range of material and geometric values considered herein. The proposed approximations, Eq. (16) and (17), do not diverge significantly from the analytical solution over the range of λ. This is because the proposed design equations as well as the analytical solutions approach an asymptote as $\lambda \rightarrow \infty$. Regardless, the design equations could be optimized for any range of λ deemed necessary for the desired application.

Although the assumption of incompressibility and inextensibility may be satisfactorily made in some cases, it is evident that it may also result in unacceptably high error in other cases. Currently the CHBDC provides no guidance on practical limits of the provisions for shear strain due to rotation or compression. This should be incorporated to avoid inappropriate application of the equations. For example, if a maximum error of 5% is permitted from the analytical solution, Eq. (17) should be used if $\lambda \geq 1.1$, otherwise incompressibility and inextensibility can satisfactorily be assumed.

In general, it is expected that designers will have more control over the extensibility of the reinforcement than the bulk modulus (e.g. the material properties and geometry of the fiber is easier to control than the bulk modulus of the elastomer). As demonstrated in [19], the extensibility of the reinforcement can be used to as an additional design parameter to optimize the design of the isolator. The use of fiber reinforcement is permitted in CHBDC and therefore should be accounted for in the design process. Furthermore, it has been demonstrated herein for a rectangular and circular pad, and shown in [4,5] for other geometries, that the current provisions have significant error when compared to the analytical solutions and can be improved with use of the proposed approximations.

CONCLUSIONS

The inclusion of seismic isolation provisions in Canadian codes and standards is expected to increase the application of base isolation technology within Canada. Consequently, it is necessary to develop increasingly comprehensive codes and standards to cover a broader range of applications and to release some assumptions that may result in serious and potentially non-conservative error. Current design equations for reinforced elastomeric bearings often neglect the compressibility of the elastomer and extensibility of the reinforcement. Although these assumptions are appropriate for bearings that have low shape factors, the use of current provisions in applications with higher shape factors may result in serious and often unconservative error. In this paper, the procedure for mathematically derived generalized approximations for critical design properties of reinforced elastomeric bearings was presented. The proposed generalized approximations were optimized for a range of compressibility and extensibility. The result is a series of equations that are appropriate for use in design guidelines that include the compressibility of the elastomer and extensibility of the reinforcement. The equations can easily be adapted to different pad geometries with tabulated coefficients. It was demonstrated that the proposed approximations are accurate over a wide range of material and geometric properties. It is recommended that these equations be extended to include annular and rectangular pad geometries. It is also recommended that committees consider the adoption of these equations into appropriate guidelines and standards or state the current range of shape factor that the existing equations are applicable for to avoid miss-use.

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